SAVI Conference
July 2026

Betting on Bets:
Anytime-Valid Tests for Stochastic Dominance

Sebastian Arnold, CWI Amsterdam
Yo Joong "YJ" Choe, INSEAD

Sebastian Arnold*
CWI Amsterdam

YJ Choe*
INSEAD

Marco Scarsini
LUISS

Ilia Tsetlin
INSEAD

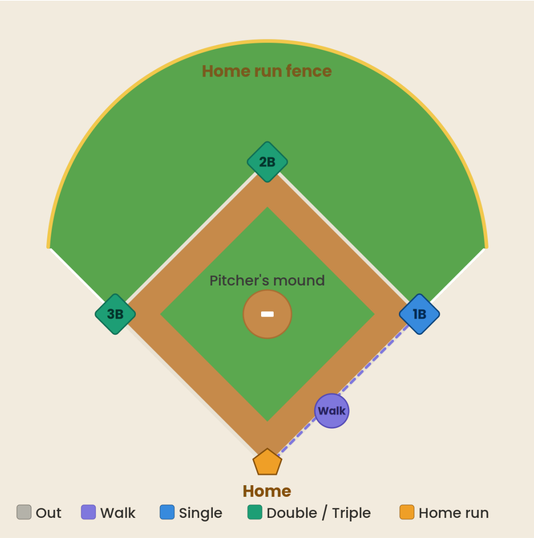
**Equal contribution.*

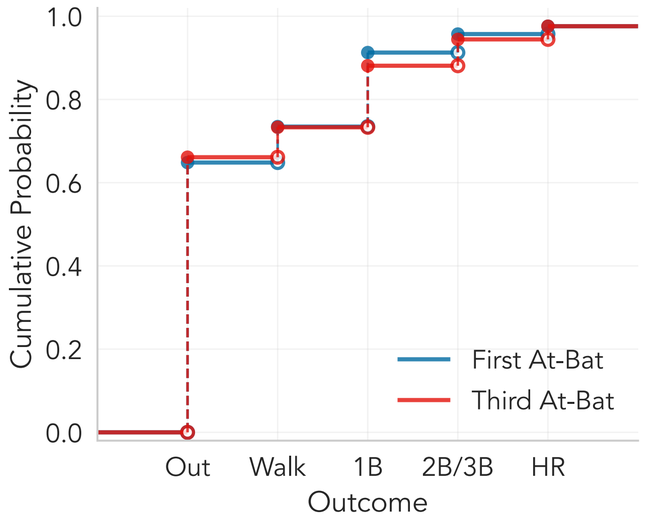
Motivation

Motivation #1: Does one prospect have an **upside** over another?

⚾ Comparing player performances:

Do batters have an upside when facing the same pitcher for the third time?





💵 Testing distributional effects:

Is there any upside in future incomes to veteran status? (Plot: Abadie, 2002)

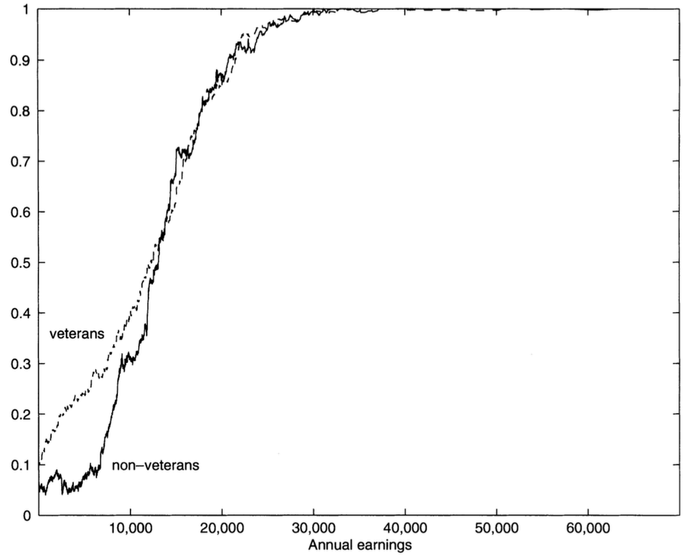


Figure 2. Estimated Distributions of Potential Earnings for Compliers.

These examples motivate **distributional** comparisons of random variables, beyond just their means, in different regions of the support.

Motivation #2: Can I monitor this test in real time?

In reality, these are *sequential* inference problems (literally, game time decisions!)

The Manager, the Ace and a Decision That Will Haunt the Rays

Blake Snell was dominating the Dodgers in a must-win World Series game. Kevin Cash still pulled him, raising painful questions of what might have been.

Blake Snell was taken out of Game 6 in the sixth inning despite having allowed just two hits. The Rays went on to lose, 3-1, ending the World Series. Kevin Jairaj/USA Today Sports, via Reuters

We are motivated to **sequentially** test/monitor for upsides, with validity at data-dependent stopping times.

The Goal

Develop **anytime-valid** methods for testing **stochastic dominance** that can be monitored for upsides at adaptive sample sizes.

...without sacrificing statistical power.

Stochastic Dominance

(a.k.a. Stochastic Ordering)

First-order stochastic dominance (1-SD)

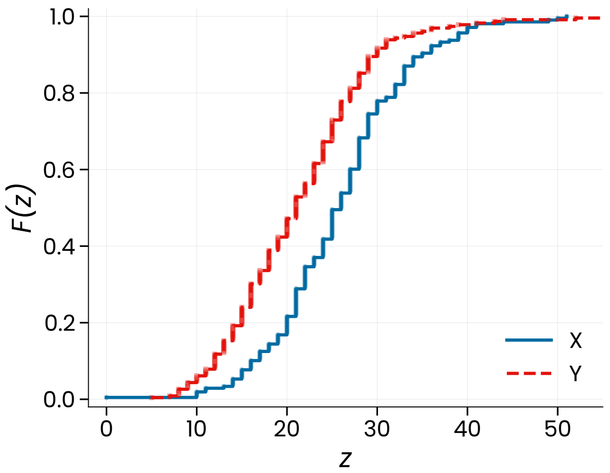
- Let X and Y be random variables with CDFs F_X and F_Y.
- **Defn:** X *stochastically dominates* Y *in the first order* if the CDF of X is entirely below the CDF of Y:

$$Y \preceq_1 X \iff F_X(z) \leq F_Y(z), \forall z$$

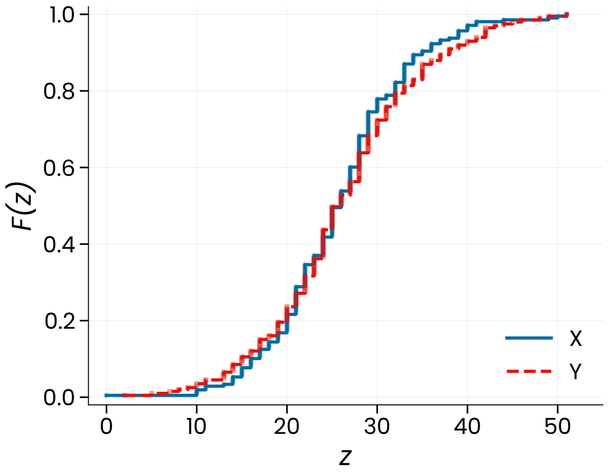
- **Expected utility view:** $Y \preceq_1 X$ *if and only if*

$$\mathbb{E}[u(Y)] \leq \mathbb{E}[u(X)]$$

for every increasing function u.



$Y \preceq_1 X$



$Y \not\preceq_1 X$

Testing for upside = Testing the 1-SD null

- Suppose we observe pairs of observations $(X_1, Y_1), (X_2, Y_2), \ldots \sim \mathbb{P}_{XY}$.
- Define the (nonparametric and composite) null hypothesis that X dominates Y:

$$H_0 = \{\mathbb{P} : Y \preceq_1 X \text{ under } \mathbb{P}\} \quad \text{vs.} \quad H_1 = \{\mathbb{P} : Y \not\preceq_1 X \text{ under } \mathbb{P}\}$$

- The null hypothesis H_0 says that Y **has no upside over** X over the entire support ($z \in \mathbb{R}$).
- **Rejecting $H_0 \equiv$ there is an upside with which Y has an advantage over X.**

E-values & intersection nulls

- Given n data points $X_1, \ldots, X_n$ and a null hypothesis H_0 (possibly composite), an **e-value** $E = E_n(X_1, \ldots, X_n)$ is any non-negative random variable satisfying:

$$\mathbb{E}_{H_0}[E] \leq 1.$$

- **For *intersection* nulls, e-values can be combined easily under arbitrary dependence:** Given e-values $E^{(z)}$ for $H_0^{(z)}$, their **(weighted) average** is an e-value for $H_0 = \cap_{z \in Z} H_0^{(z)}$:

$$\mathbb{E}_{H_0}\left[\sum_{z \in Z} w^{(z)} E^{(z)}\right] = \sum_{z \in Z} w^{(z)} \mathbb{E}_{H_0}\left[E^{(z)}\right] \leq 1.$$

Betting on bets:
An optimal e-variable for testing SD nulls

First step: Decompose the null hypothesis

$$H_0 = \{\mathbb{P} : Y \preceq_1 X \text{ under } \mathbb{P}\} = \{\mathbb{P} : \mathbb{P}(X \le z) \le \mathbb{P}(Y \le z), \forall z\}.$$

- Highly composite family of distributions. However, it is an intersection null:

$$H_0 = \bigcap_z H_0(z), \quad \text{where} \quad H_0(z) = \{\mathbb{P} : \mathbb{P}(X \le z) \le \mathbb{P}(Y \le z)\}.$$

- If we have an e-value $E(z)$ for each $\mathscr{H}_0(z)$, then **any mixture** over z is an e-value:

$$E = \int E(z) d\psi(z) \text{ is an e-value for } H_0.$$

The building-block GRO e-value

- For each "test threshold" $z \in \mathbb{R}$, let $D(z) = \mathbf{1}(X \le z) - \mathbf{1}(Y \le z)$, and consider

$$\mathscr{H}_0(z) = \{\mathbb{P} : \mathbb{P}(X \le z) \le \mathbb{P}(Y \le z)\} = \{\mathbb{P} : \mathbb{E}_{\mathbb{P}}[D(z)] \le 0\}\,.$$

Lemma.

(a) For any bet $\lambda \in [0, 1]$, $S(\lambda, z) = 1 + \lambda D(z)$ is an **e-value** for $\mathscr{H}_0(z)$.

(b) For each alternative $\mathbb{Q} \notin \mathscr{H}_0(z)$, there is a **growth-rate optimal (GRO)** bet:

$$\lambda^\star(z) = \frac{\mathbb{Q}(X \le z < Y) - \mathbb{Q}(Y \le z < X)}{\mathbb{Q}(X \le z < Y) + \mathbb{Q}(Y \le z < X)} = \frac{\mathbb{Q}(X \le z) - \mathbb{Q}(Y \le z)}{\mathbb{Q}(X \le z < Y) + \mathbb{Q}(Y \le z < X)} > 0\,.$$

- ***Growth rate:*** $g_z(\lambda) = \mathbb{E}_{\mathbb{Q}}[\log S(\lambda, z)] = \mathbb{Q}(D(z) = 1) \cdot \log(1 + \lambda) + \mathbb{Q}(D(z) = -1) \cdot \log(1 - \lambda)\,.$

Handling the intersection hypothesis

- Now that we have an optimal e-value for each threshold $z \in Z$, we can take any weighted average to obtain a valid e-value for the SD null $\mathscr{H}_0$ (intersection over z's).

$$E = \int_Z E(z)\mathrm{d}\psi(z), \text{ for any mixture distribution } \psi \text{ on } Z.$$

- Then, across multiple rounds of data (sequentially observed), we can simply **multiply** these mixtures of GRO e-values!

Main result: A powerful e-process & a test of power one for SD

- At $t \in \mathbb{N}$, let $\hat{\lambda}_t^\star(z)$ the (empirical) predictable plug-in of $\lambda_t^\star(z)$ using the first $(t-1)$ obs.

- Define the mixture process with a predictable *mixture* $(\psi_t)_{t\in\mathbb{N}}$ over z:

$$E_t = \prod_{\ell=1}^{t} S_\ell, \text{ where } S_\ell = \int_Z S(\hat{\lambda}_\ell^\star(z), z)\, d\psi_\ell(z).$$

Theorem (Anytime-validity and power of the GRO e-process).

(a) $(E_t)_{t\in\mathbb{N}}$ is an e-process for the 1-SD null H_0.

(b) For reasonable* choices of $(\psi_t)_{t\in\mathbb{N}}$, the e-process is powerful under any alternative $\mathbb{Q}$:

$$\mathbb{Q}\left(\liminf_{t\to\infty} \frac{1}{t}\log(E_t) > 0\right) = 1, \text{ and the resulting sequential test has asymptotic power one.}$$

**Eventually assign enough mass on non-SD region: $\psi_t(\{z : \mathbb{Q}_X(z) - \mathbb{Q}_Y(z) > \varepsilon\}) > \delta$ for large t.*

Choosing the predictable mixture weights

- For each threshold, we already have a GRO e-value. So, we want to make sure that the mixture weights is adaptive and do not "miss" any important (non-dominance) regions.

- The simplest choice,

$$\psi_t = \psi = \frac{1}{m}\sum_{i=1}^{m} \delta_{z_{i'}}$$

  can work well if $\{z_1, \ldots, z_m\}$ "sufficiently cover" the non-dominance regions of interest.

- When the support is unbounded, we recommend adjusting the m thresholds to the empirical quantiles of observed data, and use *exponential weights* adapted to those quantiles:

$$\psi_t = \sum_{i=1}^{n_t} w_t(z_i^t)\delta_{z_i^{t'}} \text{ where } w_t(z_i^t) \propto \exp\left(\eta \cdot \frac{\hat{F}_X^{t-1}(z_i^t) - \hat{F}_Y^{t-1}(z_i^t)}{\hat{\sigma}_{t-1}}\right)$$

*Why "betting on bets"?

- Consider two uncertain outcomes X and Y, say the daily returns of two financial assets.
- Fix a "test threshold" (say, $z = 0\,\%$). Bookmaker proposes a ***double-or-nothing-or-push bet*** with the following rules.
  - *If Y nets a positive return ($Y > z$) but X does not, then you **double** your bet.*
  - *If X nets a positive return ($X > z$) but Y does not, then you **lose** your bet.*
  - *Otherwise, nothing happens **("push")**.*
- *(Bookmaker hypothesis = $H_0(z)$ = "Probability of Y greater than z is no better than of X.")*
- Skeptic places a **fraction** of her money on this bet.

*Over repeated rounds, Skeptic's wealth is an **e-process** for "upside" (SD) testing.*
(even when Skeptic can split money across different test thresholds!)

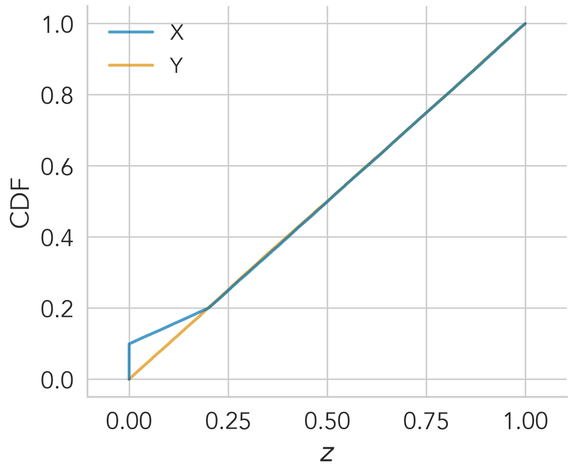
Simulations

Baselines

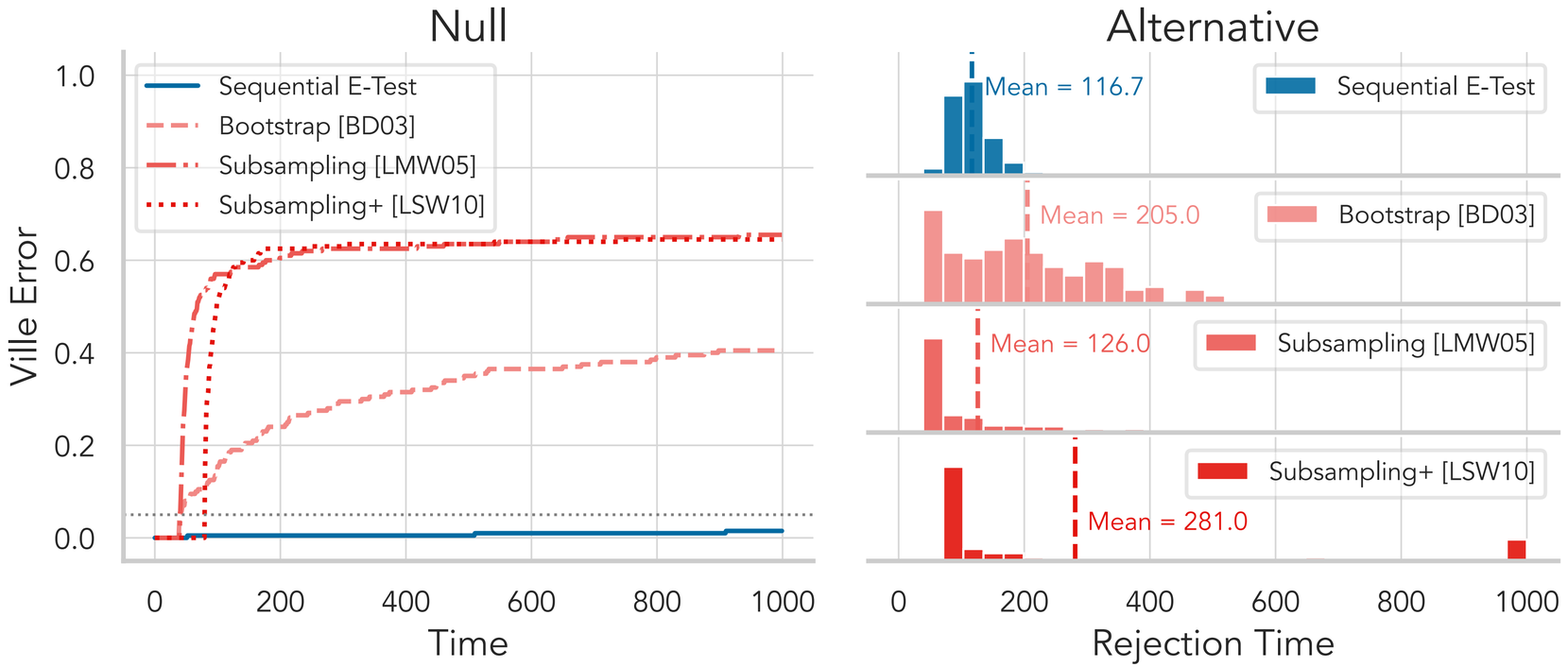
Denote the difference $\Delta_{t-1}(z) = \sum_{\ell=1}^{t-1} D_\ell(z) = t\left[\hat{F}_X^{t-1}(z) - \hat{F}_Y^{t-1}(z)\right]$, and $V_{t-1}(z) = \sum_{\ell=1}^{t-1} D_\ell^2(z)$.

- **AdaGRO-Exp:** exponential weights with self-normalization; $w_t(z) \propto \exp\left\{\eta \cdot \Delta_{t-1}(z)/\sqrt{V_{t-1}(z)}\right\}$
- **AdaGRO-Linear:** linear weights using GRO bets themselves; $w_t(z) \propto \hat{\lambda}_t^\star(z)$
- **AdaGRO-Greedy** (Shekhar & Ramdas, 2023): all-in on the max; $w_t(z) = 1\left\{z = \text{argmax}_z\, \Delta_{t-1}(z)\right\}$
- **GRO:** non-adaptive GRO baseline with simple averaging; $\psi_t = \psi = \frac{1}{m}\sum_{i=1}^{m} \delta_{z_i}$
- **Constant:** non-adaptive, constant-bet (non-GRO) baseline; $\lambda_t(z) = 0.1$

Simulation #1: Comparison with classical, non-SAVI tests



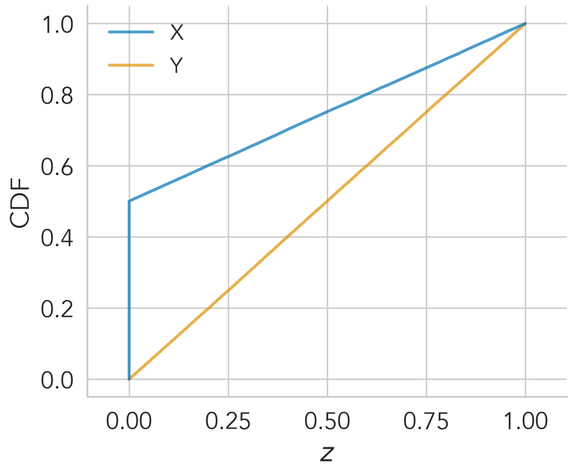
(the **"contact"** case)



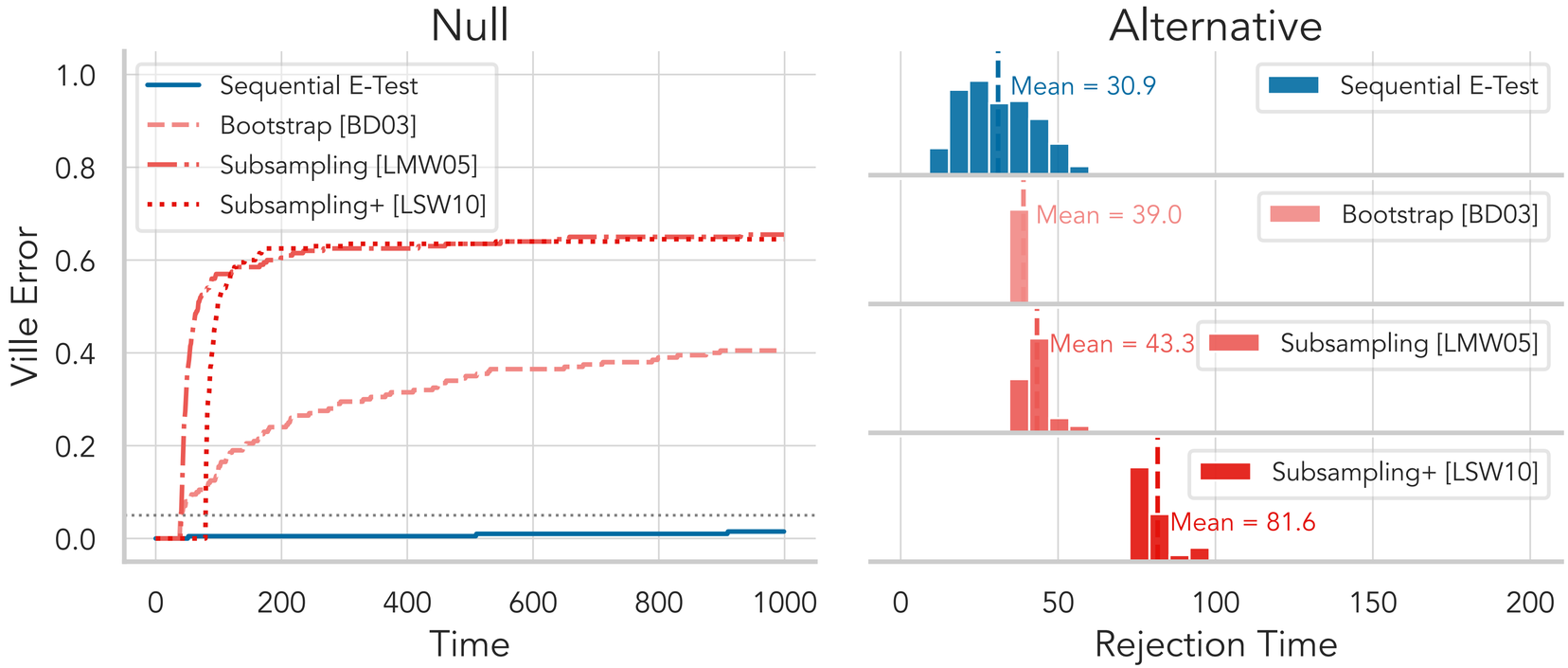
Ville Error: $\hat{\mathbb{P}}(\exists t : E_t \geq 1/\alpha)$

Rejection Time: $\tau_\alpha = \inf\{t : E_t \geq 1/\alpha\}$

Simulation #1: Comparison with classical, non-SAVI tests



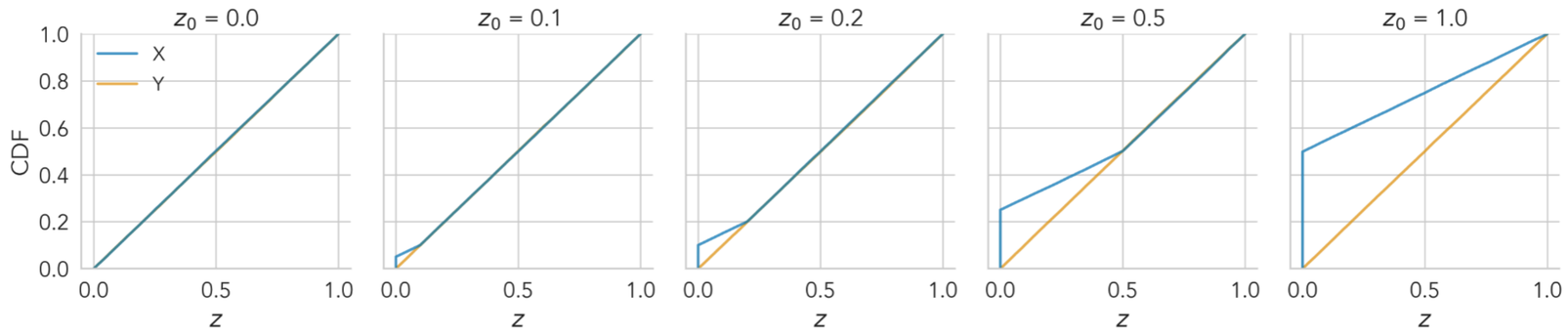
(the **"no-contact"** case)



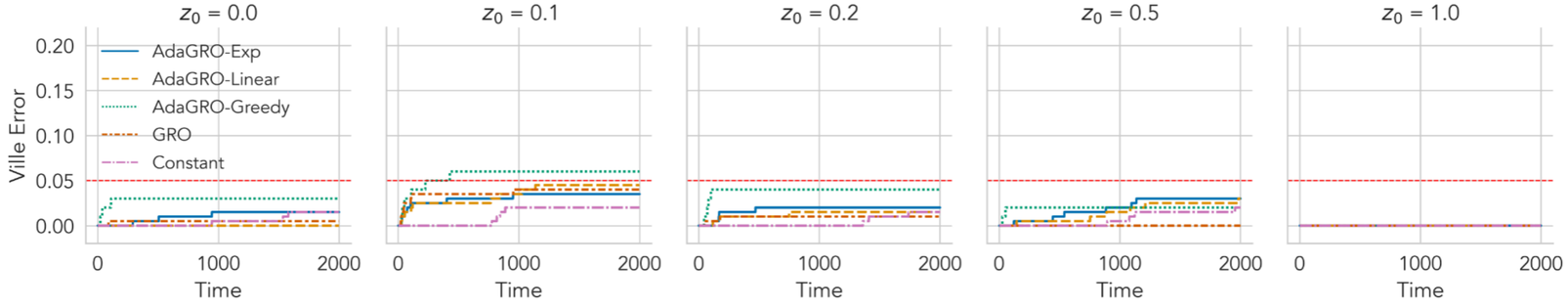
Ville Error: $\hat{\mathbb{P}}(\exists t : E_t \geq 1/\alpha)$

Rejection Time: $\tau_\alpha = \inf\{t : E_t \geq 1/\alpha\}$

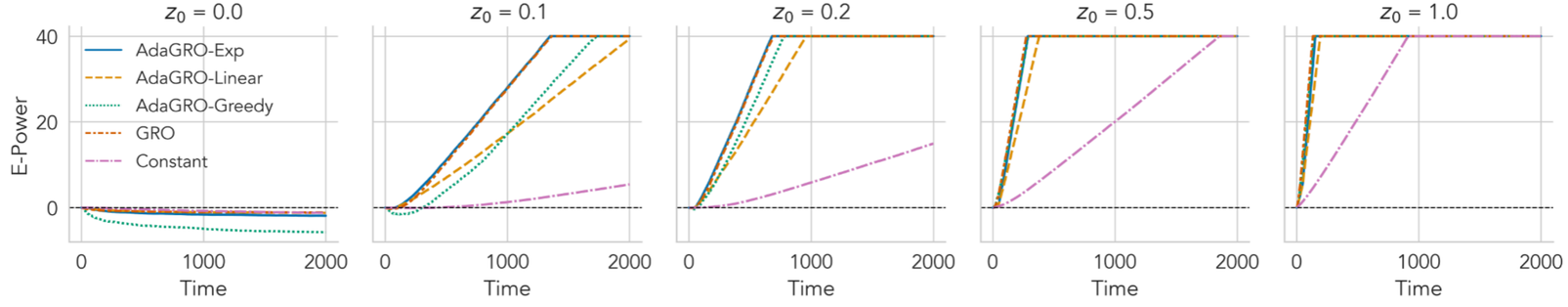
Simulation #1': Robustness to "contact sets" between CDFs



(a) CDFs of X and Y for $z_0 \in \{0.0, 0.1, 0.2, 0.5, 1.0\}$.

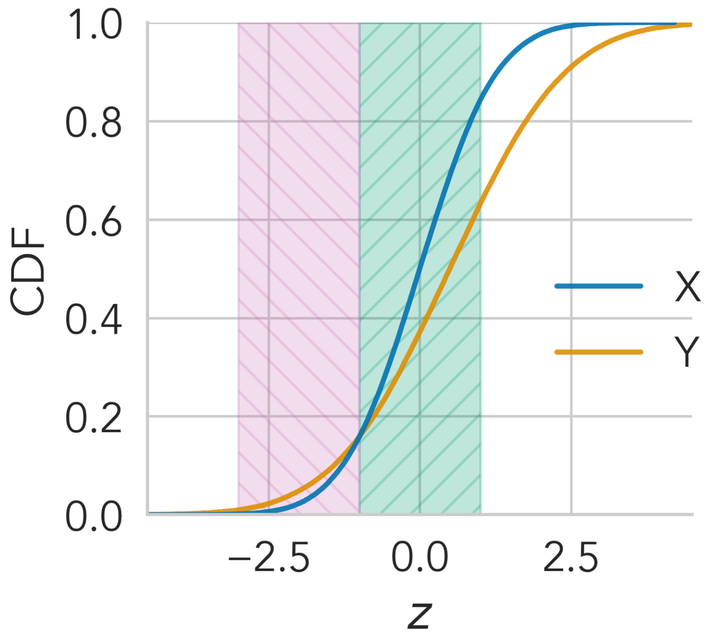


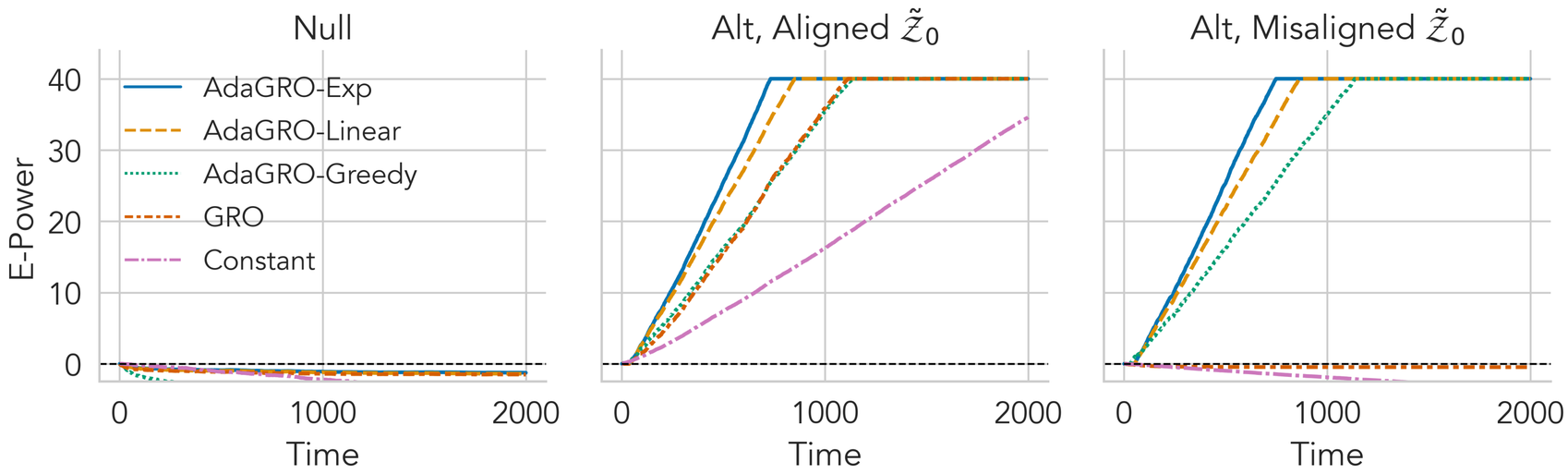
(b) Ville error for testing $\mathcal{H}_0 : X \preceq_1 Y$.



(c) E-power against $\mathcal{H}_0 : Y \preceq_1 X$.

Simulation #2: Adaptivity to non-dominance regions





Application: The third time-through-the-order penalty in baseball

Examining the 3rd time-through-the-order (3TTO) penalty

The Manager, the Ace and a Decision That Will Haunt the Rays

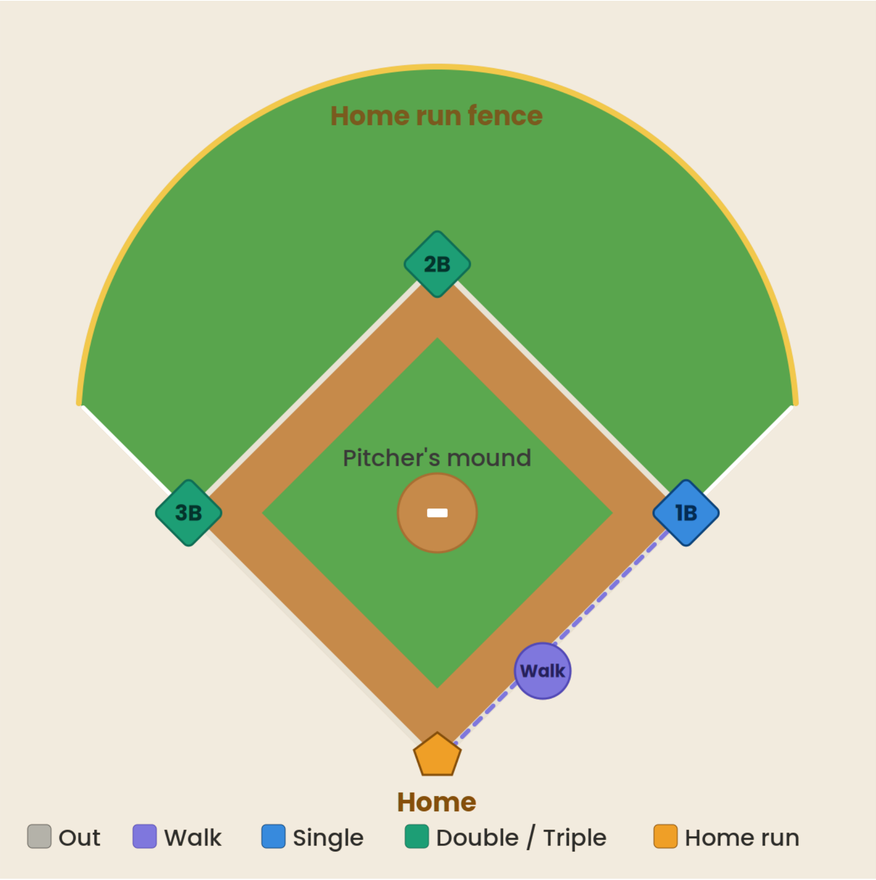
Blake Snell was dominating the Dodgers in a must-win World Series game. Kevin Cash still pulled him, raising painful questions of what might have been.

Blake Snell was taken out of Game 6 in the sixth inning despite having allowed just two hits. The Rays went on to lose, 3-1, ending the World Series. Kevin Jairaj/USA Today Sports, via Reuters

A starting pitcher typically faces the same lineup of nine batters multiple times in a game.

Should the manager replace the pitcher before batters face him for the third time?

⚾ Examining the 3rd time-through-the-order (3TTO) penalty

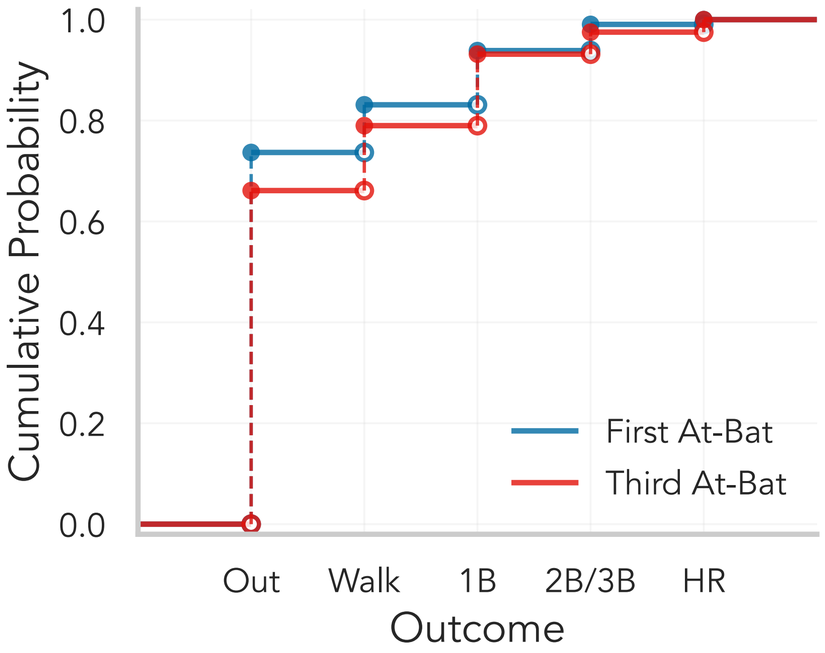


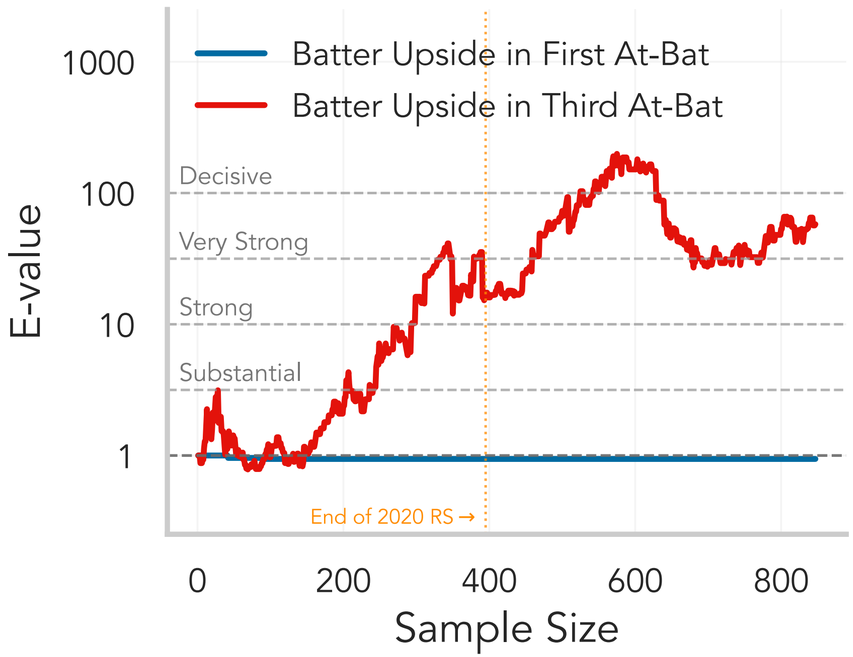
PC: Claude 4.8

Data: paired at-bats for MLB pitcher Blake Snell (2016–2025).
X: outcome in 1TTO. **Y:** outcome in 3TTO.
Order: Out < Walk < 1B < 2/3B < HR.

H0: No upside for batters in 3TTO.
H1: Batters have an upside in 3TTO.

Application: Monitoring the third-time effect in baseball ⚾

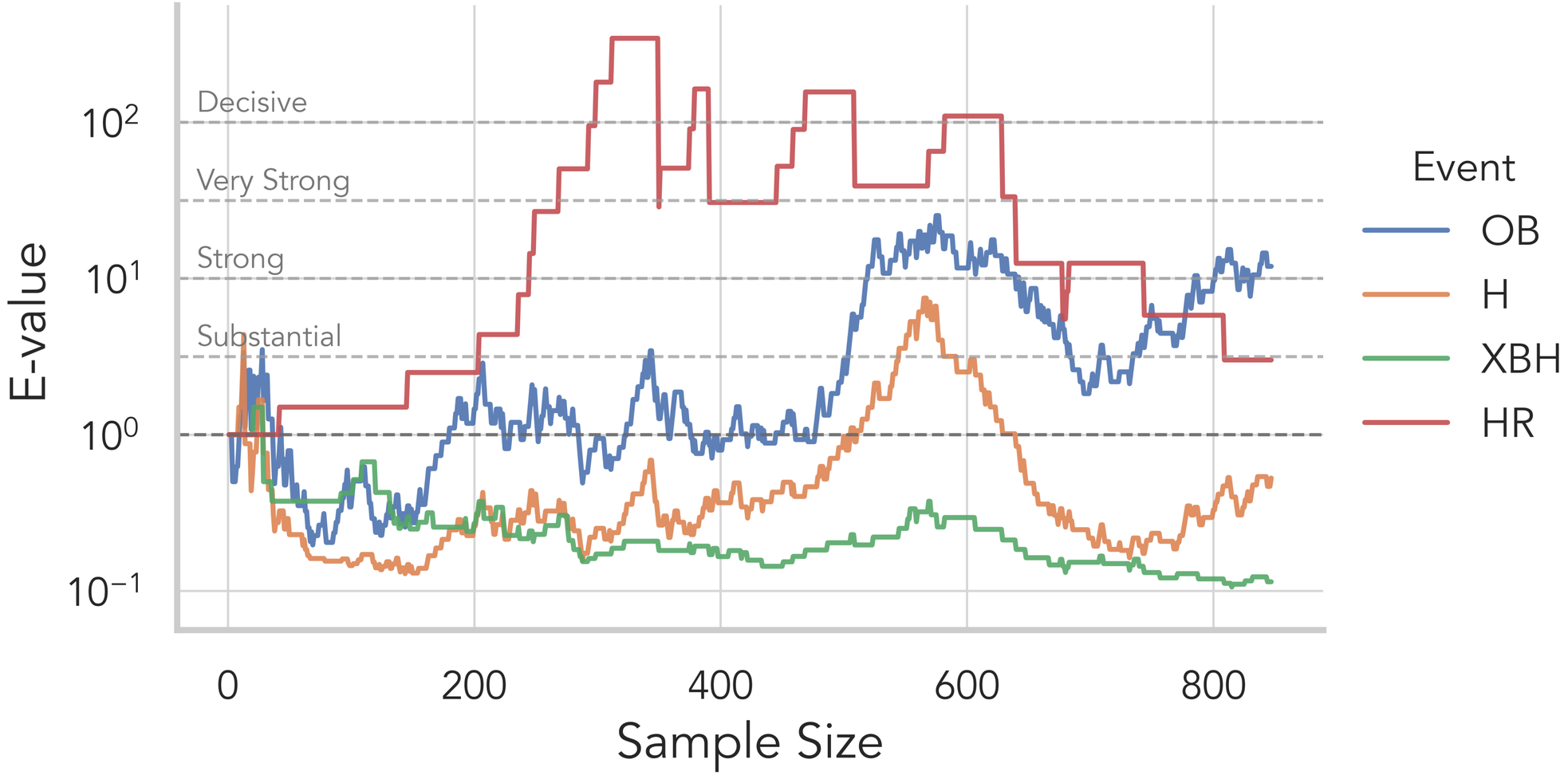




Data: Every paired at-bat outcome against MLB pitcher Blake Snell, from 2016 to 2025 (regular seasons).

Fine-grained testing for each $H_0^{(z)}$

OB (on-base): Walk, 1B, 2B, 3B, HR
H (hit): 1B, 2B, 3B, HR
XBH (extra-base hit): 2B, 3B, HR



Data: Blake Snell's regular season pitching records, 2016-2025

Affirming SD (i.e., *definite* upside): Testing the non-dominance null

An impossibility result for testing non-SD

- Suppose we want to show the *stronger* claim that Y has a **definite upside** over X.
- In this case, we want to reject the the *non-SD null (in first order)*

$$\mathscr{H}_0' = \{\mathbb{P} : \mathbb{P}(\mathsf{X} \leq \mathsf{z}) < \mathbb{P}(\mathsf{Y} \leq \mathsf{z}) \text{ for some } \mathsf{z}\} = \left\{\mathbb{P} : \mathbb{P}_{\mathsf{X}} \not\preceq \mathbb{P}_{\mathsf{Y}}\right\},$$

a ***non-convex*** union-hypothesis. The null & alternative are switched!

Corollary (No nontrivial e-variable exists for the non-SD null). There exists no e-variable E for $\mathscr{H}_0'$ which is nontrivial for all $\mathbb{Q} \in (\mathscr{H}_0')^{\mathsf{c}}$ (that is, $\mathbb{E}_{\mathbb{Q}} \log \mathsf{E} > 0$) simultaneously.

follows from Theorem 6.2 by Zhang et al. (2024)

The minimum GRO e-value (finite support)

- Assume **finite** (and known) support and add separation from the null:

$$\mathcal{Q}(\varepsilon) = \{\mathbb{Q} : \mathbb{Q}(X \leq z_i) > \mathbb{Q}(Y \leq z_i) + \varepsilon, \text{ for all } z = z_1, \ldots, z_{m-1}\}.$$

Proposition (Validity and asymptotic optimality of the min-approach)

(a) $S = \min_{i=1,\ldots,m-1} S(\lambda_i, z_i)$ is an e-value for $\mathcal{H}'_0$, for any bet $\lambda_i \in [0, 1]$,

(b) For the predictable GRO plug-in, $\mathbf{e}_t = \min_{i=1,\ldots,m-1} \left(\prod_{\ell=1}^{t} S_{i,\ell} \right) \to \infty$, for any $\mathbb{Q} \in \mathcal{Q}(\varepsilon)$,

and we have the asymptotically optimal growth-rate: $\liminf_{t\to\infty} \frac{1}{t}(\log \mathbf{e}_t - \log E_t) \geq 0$.

Affirming FSD for continuous outcomes

- Continuous CDFs F_X and F_Y on Z will *always* cross at the boundaries/tails.

- Thus, we have to **restrict** the domain of interest to $\tilde{Z} \subseteq Z$ (Davidson and Duclos, 2013).

- Again, consider *separated* (and *restricted)* alternative

$$\mathcal{Q}(\varepsilon, \tilde{Z}) = \{\mathbb{Q} : \mathbb{Q}(X \leq z_i) > \mathbb{Q}(Y \leq z_i) + \varepsilon, \text{ for all } z \in \tilde{Z}\}.$$

Proposition. We can construct sequential tests for the non-SD null that achieve *asymptotic power one* against alternatives in $\mathbb{Q} \in \mathcal{Q}(\varepsilon, \tilde{Z})$.

- These tests are based on *time-uniform CDF bands*.
  Howard and Ramdas (2022), Mineiro and Howard (2023), and Clerico et al (2026).

Conclusion

Conclusion

- **We developed a novel family of e-processes & sequential tests for various SD notions.**
  - *These methods are fully nonparametric and robust to dependence between X and Y.*
  - *The e-based approach performs competitively in power with classical, non-SAVI methods.*
- **We discuss potential extensions to non-i.i.d., unpaired, and multivariate data.**
- **We also discuss extensions to general, integral stochastic orders.**
  - Notable examples: increasing convex orders (i.e., risk-seeking DMs) & infinite-order SD.
- **Non-SD null testing remains challenging, and there are various extensions for future work.**
  - In particular, affirming weaker notions SD (higher-order SD; almost SD; …) remains hard.

Thank you!

Any questions?

https://arxiv.org/abs/2604.21851

Appendix

Testing higher-order SD

Higher-order SD

- More generally, $\mathbf{X}$ *stochastically dominates* $\mathbf{Y}$ *in the* $\mathbf{k}$*-th order* if

$$\mathbb{E}[u(Y)] \leq \mathbb{E}[u(X)] \text{ for every "utility" function } u \in \mathcal{U}^{[k]}.$$

- *E.g.*, 2nd order utility class $\mathcal{U}^{[2]}$ consists of all **increasing & concave** functions (risk-averse DMs).
- Generally, the $\mathbf{k}$-th order utility class consists of functions that alternate signs in their first $\mathbf{k}$ derivatives.
  - 3rd order models "prudent" DMs (marginal utility is convex): implies positive skew
  - 4th order captures "temperate" DMs (reluctance to accept additional risk): implies thinner tails
- **Testing for higher-order SD = Testing for upside by a DM with a partially specified utility function.**

Characterizing k-th order SD with generators

- Formally, X *stochastically dominates* Y *in the* k*-th order* **(k-SD)**, or $Y \preceq_k X$, if

$$F_X^{[k]}(z) \le F_Y^{[k]}(z), \quad \forall z \in \mathbb{R}, \quad \text{where} \quad F^{[k]}(z) = \int_{-\infty}^{z} F^{[k-1]}(u)du.$$

(we set $F^{[1]} \equiv F$).

Lemma (Characterizations of k-SD). The following statements are equivalent:

(a) $Y \preceq_k X$, or $F_X^{[k]}(z) \le F_Y^{[k]}(z)$ for all $z \in \mathbb{R}$.

(b) [Utility View] $\mathbb{E}[u(Y)] \le \mathbb{E}[u(X)]$ for any k-th order utility function $u \in \mathscr{U}^{[k]}$.

(c) [Generator View] $\mathbb{E}[(z - X)_+^{k-1}] \le \mathbb{E}[(z - Y)_+^{k-1}]$ **for all** $z \in \mathbb{R}$**.**

Extension via the integral identity

- Proof of the generator characterization is based on the following identity (via Fubini):

$$F^{[k]}(z) = \frac{1}{(k-1)!}\mathbb{E}\left[(z - X)_+^{k-1}\right].$$

- Thus, we can rewrite **the k-SD null hypothesis** as another intersection null over **z**!

$$\mathscr{H}_0^{[k]} = \bigcap_{z \in \mathbb{R}} \mathscr{H}_0^{[k]}(z), \quad \text{where} \quad \mathscr{H}_0^{[k]}(z) = \{\mathbb{P} : \mathbb{E}_{\mathbb{P}}[(z - X)_+^{k-1}] \leq \mathbb{E}_{\mathbb{P}}[(z - Y)_+^{k-1}]\}.$$

Main result: Asymptotically powerful e-processes for testing k-SD

Assumption. The support of X and Y is bounded from below: $Z = [a, \infty)$.

- Define bet $S^{[k]}(\lambda, z) = 1 + \lambda[u_z(Y) - u_z(X)]$, for the k-th order, *normalized* utility generator

$$u_z(x) = -\left[\frac{(z-x)_+}{z-a}\right]^{k-1} \in [-1, 0],\ x \in Z$$

Theorem (Anytime-validity and power for k-th order SD null).

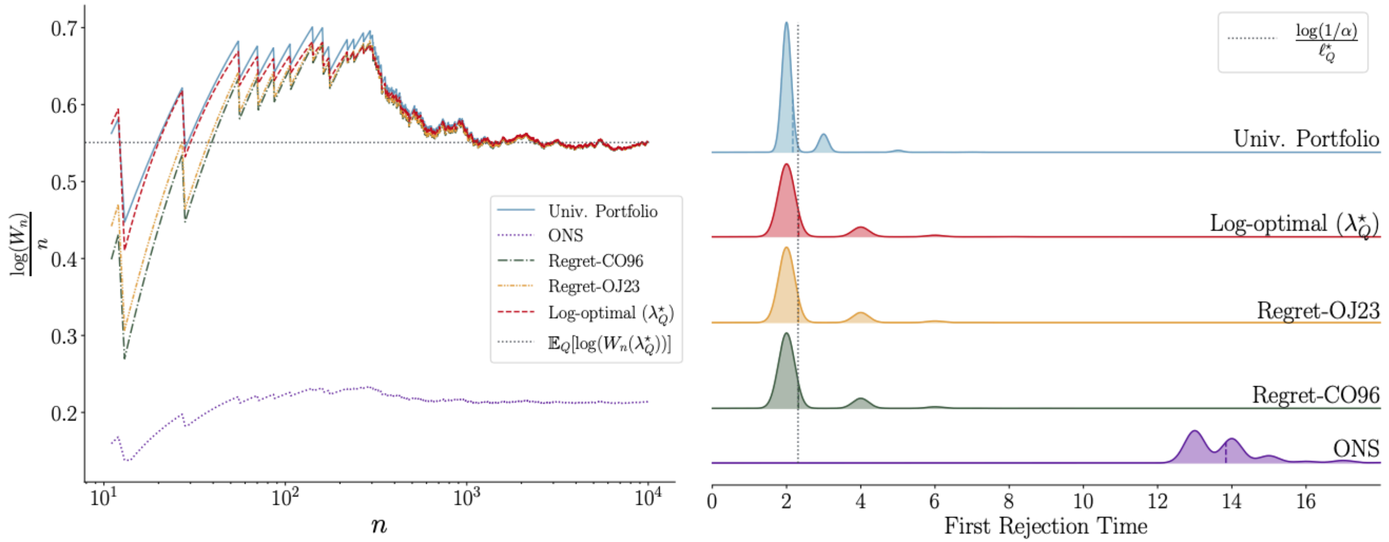
(a) For any predictable mixture $(\psi_t)_{t\in\mathbb{N}}$ and bets $(\lambda_t(z))_{t\in\mathbb{N}}$, we have an e-process for $\mathscr{H}_0^{[k]}$:

$$E_t^{[k]} = \prod_{\ell=1}^{t} S_\ell^{[k]}, \text{ where } S_\ell^{[k]} = \int_Z S^{[k]}(\lambda_\ell(z), z)\, d\psi_\ell(z).$$

(b) If we learn $(\lambda_t(z))_{t\in\mathbb{N}}$ by UP, we have again exp. growth under any alternative $\mathbb{Q}$.*

*How do we choose the bets?

- For k-SD, we don't have a growth-rate optimal (GRO) bet; bet outcome is no longer ternary.
- But we have a *bounded* outcome, and many betting strategies that we know would work.
- For each threshold **z**, we default to **universal portfolio bets $\lambda_t^{\mathrm{UP}}(z)$**, which are asymptotically log-optimal and achieves sublinear portfolio regret (Waudby-Smith et al., 2025, figure below).



Additional Materials

E-processes quantify evidence in sequential experiments

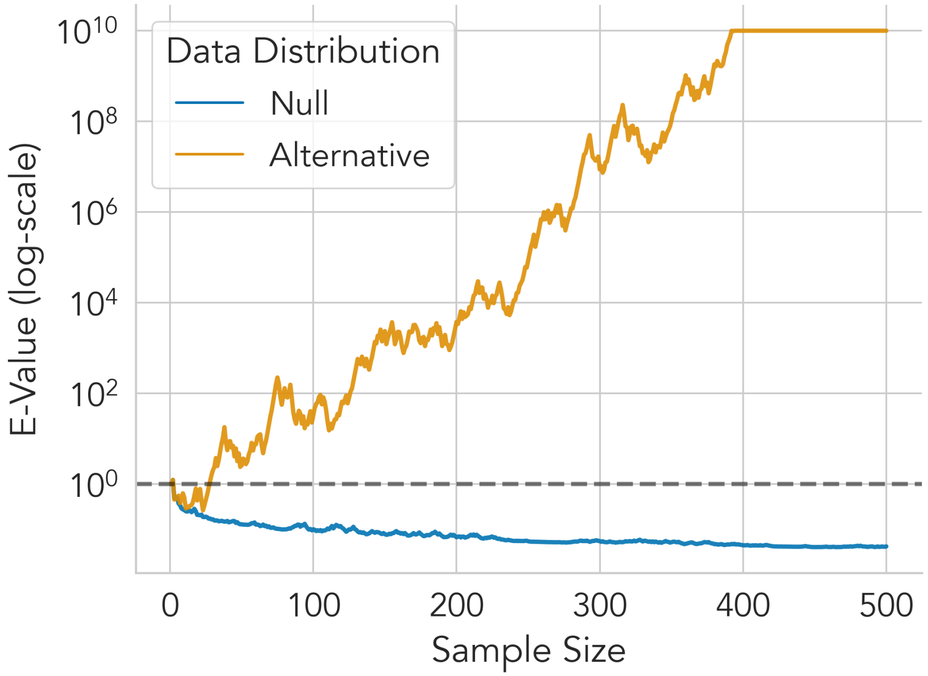
E-process $(E_t)_{t\geq 0}$

A non-negative process for H_0

For any stopping time τ,

$$\mathbb{E}_{H_0}[E_\tau] \leq 1.$$

"ANYTIME-VALIDITY"



An e-process is expected to be small under the null.

We want it to grow large under the alternative.

Ville's inequality: From e-processes to sequential tests

- Let $\alpha \in (0, 1)$ be any significance level.
- **Ville's inequality** for test martingales & e-processes:

$$P\left(\exists t \geq 1 : E_t \geq 1/\alpha\right) \leq \alpha, \ \forall \alpha \in (0, 1).$$

Jean Ville

- This is **equivalent** to a time-uniform guarantee for sequential testing:

$$P\left(\exists t \geq 1 : E_t \geq 1/\alpha\right) \leq \alpha, \ \forall \alpha \in (0, 1).$$

Alternatives based on time-uniform confidence bands

- Instead of taking mixtures or minima over z, we can also consider testing the KS parameter:

$$\theta = \sup_z [F_Y(z) - F_X(z)]_+ \leq \|F_X - F_Y\|_\infty.$$

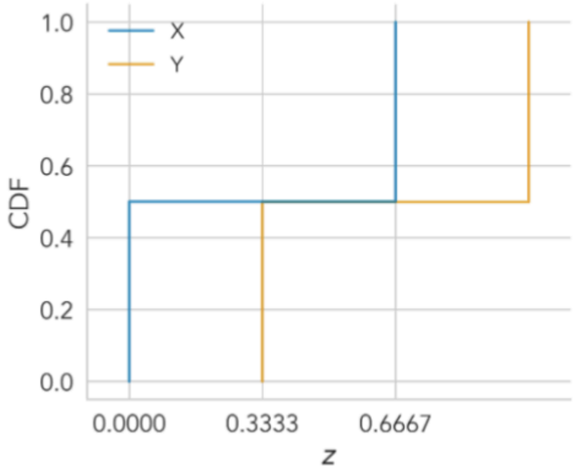
- The theory of reverse martingales yields a **time-uniform concentration* around θ,** with which we can either construct a reverse martingale or a confidence sequence around θ:

$$P(\exists t \geq 1 : \theta \in C_t^{(1-\alpha)}) \geq 1 - \alpha, \text{ where}$$
$$C_t^{(1-\alpha)} = \left(\|\hat{F}_X(t) - \hat{F}_Y(t)\|_\infty - \kappa_t, \|\hat{F}_X(t) - \hat{F}_Y(t)\|_\infty + \gamma_t \right).$$

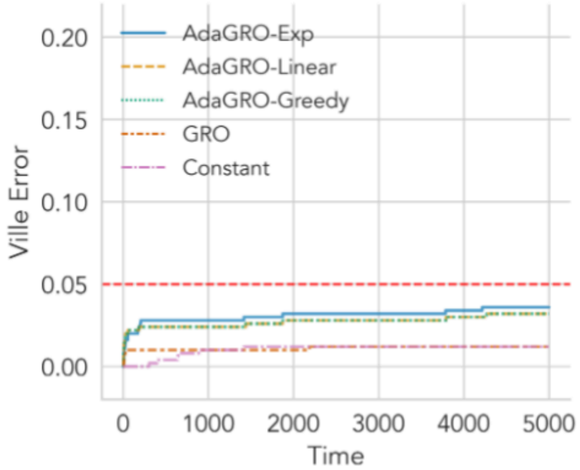
- More conservative in practice for testing SD.

**Reference: Manole & Ramdas (2023)*

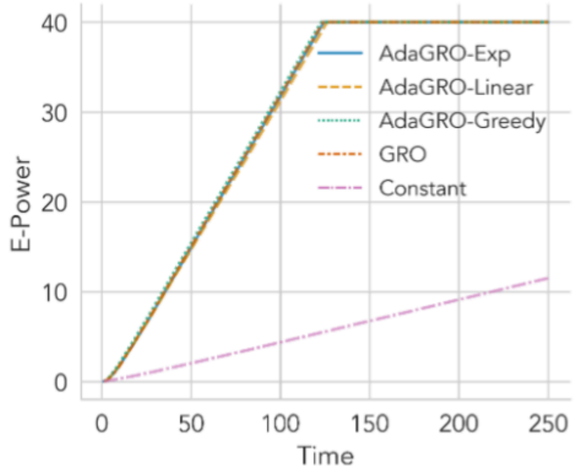
Simulation #A: Finite support with anti-monotonicity



(a) F_X and F_Y.



(b) Ville error for $\mathcal{H}_0 : X \preceq_1 Y$.



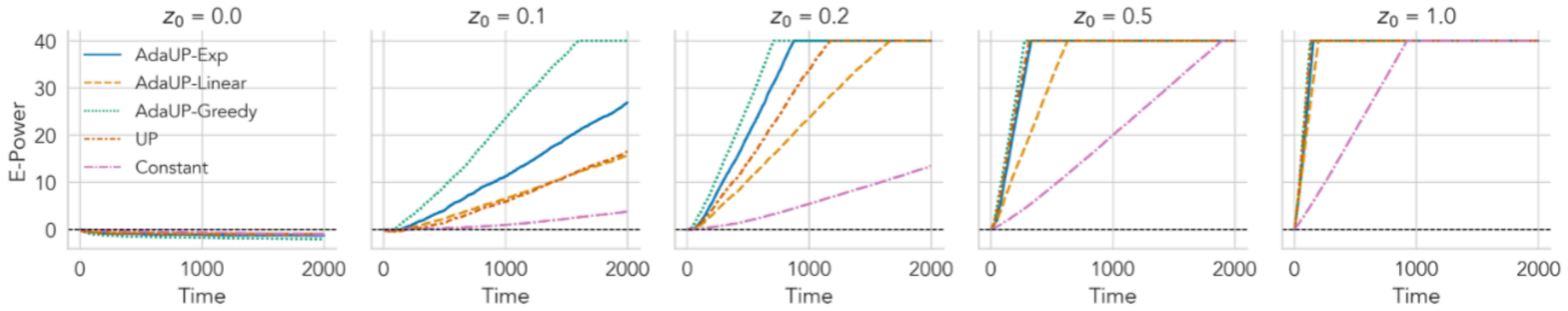
(c) E-power against $\mathcal{H}_0 : Y \preceq_1 X$.

$$\mathbb{P}(X = 0, Y = 1) = 1/2$$
$$\mathbb{P}(X = 2/3, Y = 1/3) = 1/2$$
$$(\rho(X, Y) = -1)$$

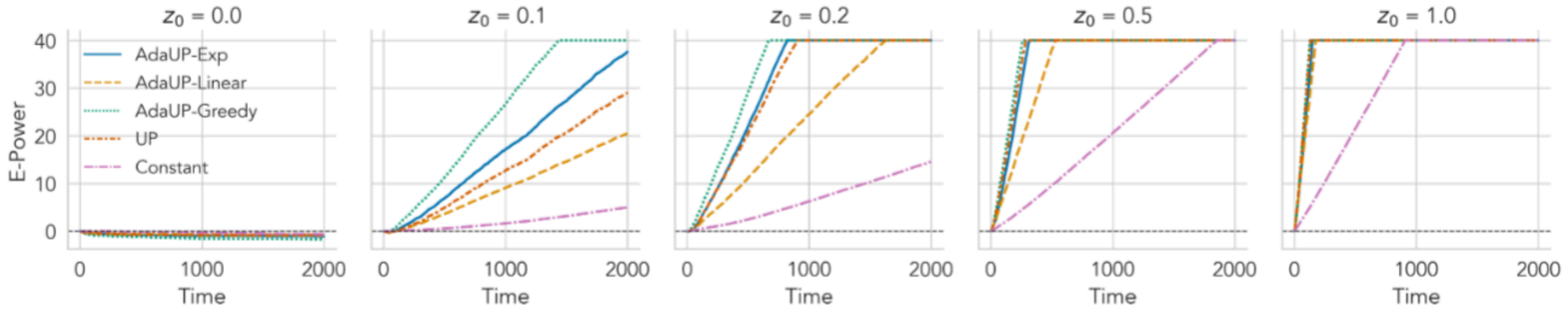
Ville Error: $\hat{\mathbb{P}}(\exists t : E_t \geq 1/\alpha)$

Rejection Time: $\tau_\alpha = \inf\{t : E_t \geq 1/\alpha\}$

Simulation #B: Testing 2-SD and 3-SD nulls

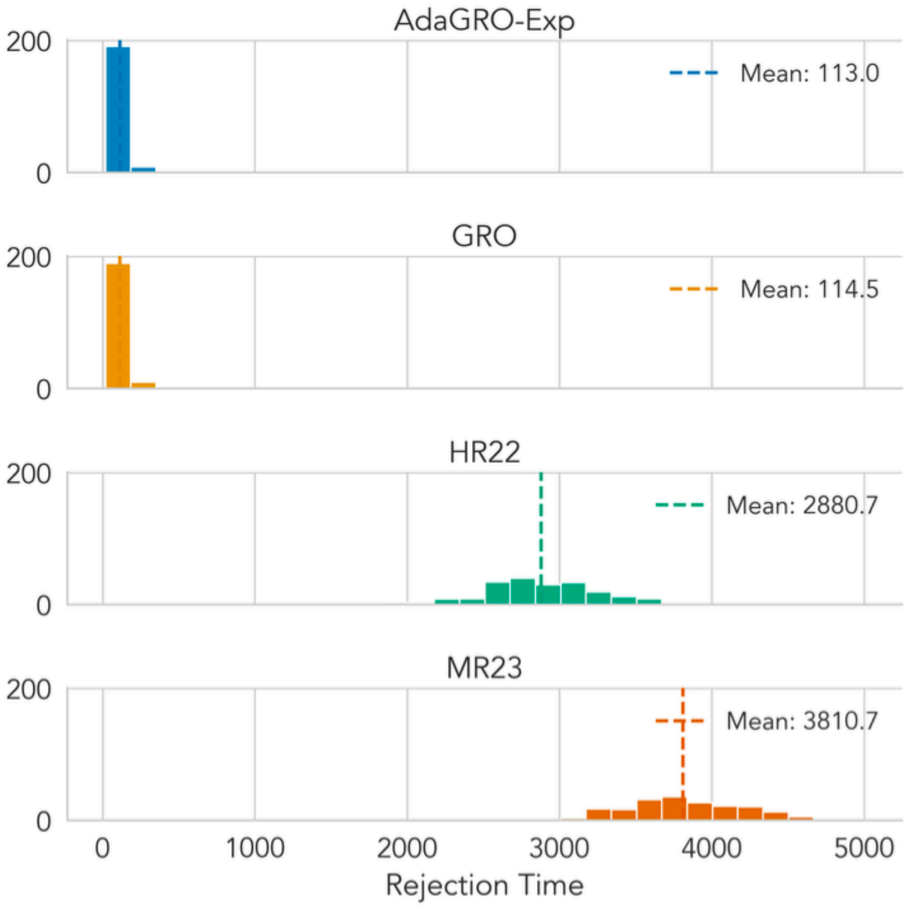


(a) E-power against $\mathcal{H}_0 : Y \preceq_2 X$ (2-SD).

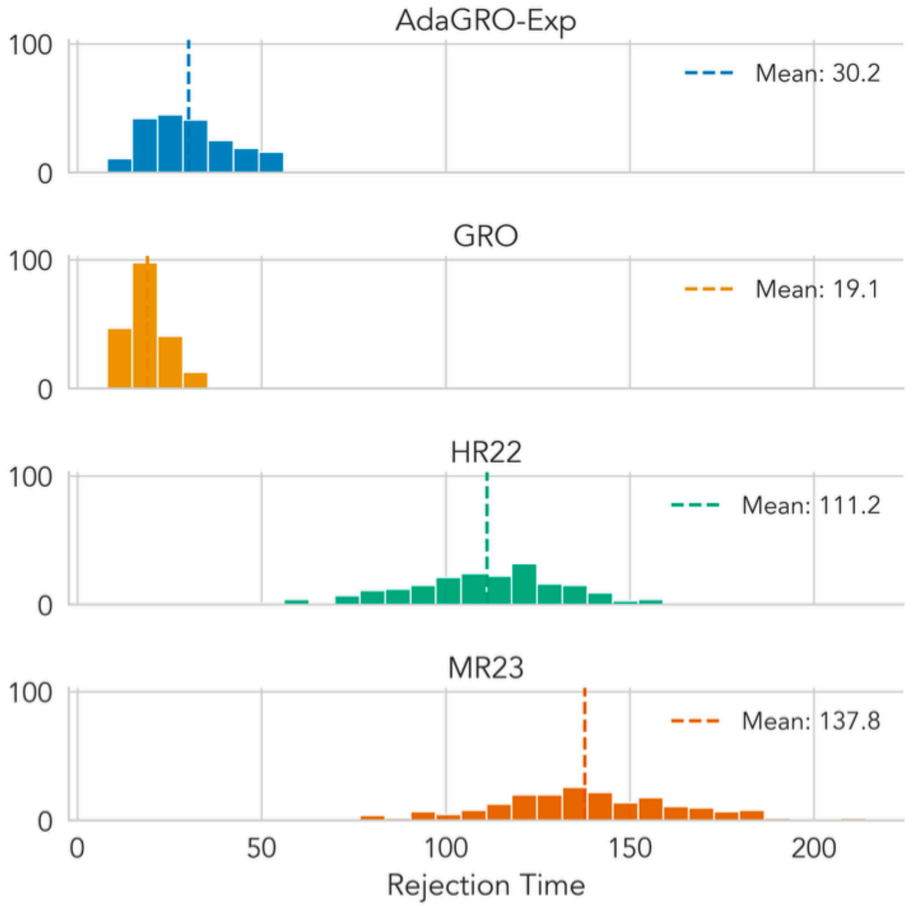


(b) E-power against $\mathcal{H}_0 : Y \preceq_3 X$ (3-SD).

Simulation #C: Comparison with time-uniform CDF bands



(a) Substantial contact between CDFs ($z_0 = 0.2$).



(b) No contact between CDFs ($z_0 = 1.0$).

The End